Metrics for Use in Quantifying Power System Resilience with Water-Energy Nexus Considerations: Mathematical Formulation and Case Study

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***Abstract*— The electric power system and the water distribution systems (WDS) have several interdependencies which have been recognized and which comprise what is informally referred to as the water-energy nexus. The need to adequately model and study these two systems and their interactions is becoming critically important due to the likelihood of load increases in both systems or due to unforeseen events. From previous studies, it is known that the power system is quite resilient under normal operating conditions as well as under small disturbances due to robust design and planning procedures. However, long-term droughts or wide spread power outages can have a severe effect and are part of a set of high impact, low probability events which need to be studied, understood and if possible, quantified. This paper presents novel performance metrics to aid in assessing the resilience of the power system when subjected to such rare, extreme conditions. The metrics proposed here are based on quantities that are relevant to the power system exclusively as well as on parameters pertaining to its operation in conjunction with the water distribution system. The resilience measures of performance are calculated using appropriate metrics which can be obtained from long-term, time-domain simulations.**

***Index Terms*— Critical Interdependent Infrastructures, Electric Power System, Extended Period Time-Domain Simulations, System Resilience, Water Distribution System, Water-Energy Nexus**

Nomenclature

$dAdj_{xf,n,t}$	Load reduction amount, $xf \in \{sf, mf\}$, bus n, time t, from consumer survey results
$LOL_{n,t}$	Loss of load, bus n, time-period t
mf_n	Fraction of multi-family (*mf*) homes in residential component of load, bus n
$P_{l,max}$	Rate A MVA limit, transmission element l
$P_{l,t}$	Active power flow, transmission element l, time-period t
$P_{load,nt}$	Real power demand, bus n, time-period t
R_{CW}^{g}	Cooling water index function, generator g
R_{DS}^{n}	Overall demand supplied index function, bus n
R_{PL}^{n}	WDS pump demand supplied index function, bus n
R_{TL}^{l}	Transmission line thermal limit index function, branch l
R_{V}^{n}	Voltage index function, bus n
$Res_{R_{CW}^{g},t}$	Resilience measure of performance for cooling water supply, time-period t
$Res_{R_{DS}^{n},t}$	Resilience measure of performance for overall demand supplied, time-period t
$Res_{R_{PL}^{n},t}$	Resilience measure of performance for WDS pump demand supplied, time-period t
$Res_{R_{TL}^{l},t}$	Resilience measure of performance for transmission lines, time-period t
$Res_{R_{V}^{n},t}$	Resilience measure of performance for voltage, time-period t
sf_n	Fraction of single-family (*sf*) homes in residential component of load at bus n
sl	Transmission line index function calculation parameter, summing length
$TL_{g,t}$	Cooling water storage tank level, generator g, time-period t
WDS	Water distribution system

I. Introduction

THE electric power system and the water distribution system (WDS) are critical infrastructures which have dependencies on each other that are clearly seen in the course

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of their normal operation [1]-[3]. The critical interdependency of the power system on the WDS is the need of water for thermoelectric power generation which is used, in the case of a closed-loop cooling cycle, mainly for cooling water make-up requirements [4]. The WDS requires electricity for the various pumps in the system which are needed to aid in the delivery of water to the consumer at sufficient pressures. A detailed framework for the modeling and control of the two systems for extreme conditions in a simulation environment suitable for long-term, time-domain simulations has been addressed [5],[6].

Increasing attention has been given to resilience calculations for various systems [7]-[9], especially given the potential benefits resulting from the enhancement of system performance in the face of extreme events [10]. Recent work in the power systems area has offered metrics and a framework by which to quantify resilience and shown the results for case studies involving weather events entailing high winds [11]. Other authors [12] have provided similar metrics and a general framework from which the overall system resilience of interdependent infrastructures can be quantified by using specified measures of performance. These measures of performance are, in general, system specific and their convenient use allows for the conducting of additional analysis regarding main aspects which determine a system's resilience.

The aspects which have been proposed for quantifying resilience [11], [12] are found by examining the behavior of the measures of performance as functions of time before, during and after system disturbances. The process of quantifying a disturbance includes measuring how low the measure of performance drops, how fast it both drops and recovers, and to what value the measure settles at. These metrics are seen to correspond directly with terms related to system resilience such as the system concepts of "robustness, response, recover, adaptability" which are common in the resilience literature [13].

II. Resilience of Power System

The resilience calculation method that is proposed in this work is done in three steps. First, an index is used to determine how the value of a specific operational quantity within the power system relates to its ideal or desired value. Second, that index is used to calculate what was called in the introduction, and following [12], a measure of performance. Because the power system is a complex system and its resilience cannot be quantified by just one system characteristic, several measures of performance are proposed in this work. The goal of these measures of performance is to relate the index function values to the overall performance of the system. Following the calculation of the desired or specified measures of performance, they are combined and normalized appropriately in order to determine the overall system resilience.

Fig. 1 shows the flow chart for the implementation of this calculation procedure. Here, it is noted that this particular analysis was implemented using post-simulation data, although the procedure could be implemented so that the calculations are done during a simulation, as applicable.

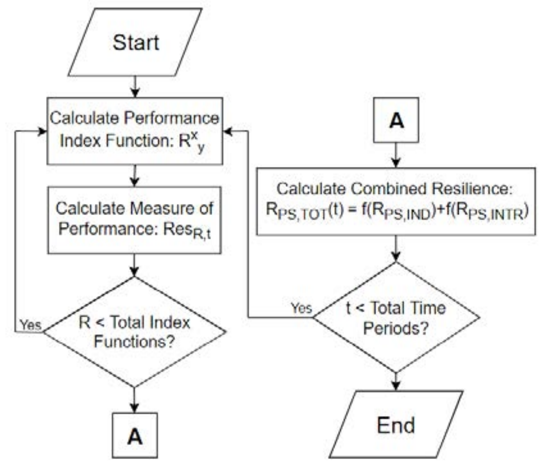

Fig. 1. Post-simulation calculation methodology and value function/measure of performance relationship.

III. Mathematical Formulation of the Proposed Metrics for Measure of Operational Resilience

The goal of this work is to quantify resilience as a weighted sum of several metrics directly related to the current and historical operating condition of the power system. These metrics are defined here in terms of quantities related to both the independent operation of the power system as well as to the interdependent operation of the electricity and water systems. These operational metrics are defined by specific functions which aim to capture the performance of the system relative to a specified quantity. The weighting of each of the terms comprising the overall resilience can be determined by a decision maker or by other means such as assigning weights according to a component's calculated structural vulnerability.

The analysis which follows is based on calculations that were done using the results from long-term simulations of the water and power systems. In these simulations, the power system network simulation was performed using time-series power flows. With this in mind, the following quantities were selected for use as operational resilience index functions:

1. Bus voltage magnitude
2. Line thermal limits
3. *Thermoelectric cooling water demand satisfaction*
4. Load (overall) supply satisfaction
5. *Load (WDS pump) supply satisfaction*

The terms in italics above denote quantities which aim to capture resilience in terms of explicit interdependencies between the two systems.

The resilience that is calculated is based on defining resilience as how well the system performs satisfactorily following unsatisfactory operation. For a given performance index function, *R,* the resilience in time period *t* is given as [14]

$$Res_{R,t} = \frac{\#\ of\ satisfactory\ operations\ following\ unsatisfactory}{\#\ of\ unsatisfactory\ operations} \quad (1)$$

Unsatisfactory operation is defined as the resilience index function being less than that function's value in the previous simulation hour (and satisfactory operation denoted by an increase in the index function's value). For the interdependent infrastructure metrics, a more restrictive definition of unsatisfactory operation in the denominator of (1) is defined as being any time-period during which the performance index function is less than its maximum value of 1. The measure of performance is defined for values between 0 and 1. The relationship between the various index functions and the resilience measures of performance is clearly seen in the flowchart of the proposed methodology, Fig. 1.

A. Bus Voltage Resilience Function

Power system bus voltage magnitudes are kept within a small range of values during normal system operation. The reason for this is that electrical equipment is designed for operation at certain rated voltages and operation under conditions which differ too greatly from the rated specifications can have adverse effects. The index function relating the operational performance of the power system to bus voltages is therefore defined to have a value of 1 for voltages within a small, nominal operating range. Outside of this range, the resilience is assumed to decrease as the deviation away from the nominal operating conditions is increased, reaching 0 at some point. Following the description above and using the quantities given in Table 1, the resilience function for bus voltage magnitude is defined mathematically as:

$$R_V^n = \begin{cases} 0 & V_n < V_{min} \\ 13.3 \cdot V_n - 12 & V_{min} \le V_n \le V_{thr,min} \\ 1 & V_{thr,min} \le V_n \le V_{thr,max} \\ -13.3 \cdot V_n + 14.7 & V_{thr,max} \le V_n \le V_{max} \\ 0 & V_{max} < V_n \end{cases} \tag{2}$$

The resilience measure of performance for bus voltages in time t of the simulation is then calculated as:

$$Res_{R_V^n,t} = \frac{\sum_{n=1}^{NB} max(0, R_V^n(t) - R_V^n(t-1))}{\sum_{n=1}^{NB} ceil(R_V^n(t-1) - R_V^n(t))} \tag{3}$$

Note that this is just an expansion of (1) in terms of the index function R_V^n.

B. Transmission Line Resilience Function

Similar to the bus voltage magnitudes in the power system, limits are imposed on the magnitude of current which flows through transmission equipment. The reason for this is that the equipment (conductors, terminating equipment such as transformers, and protection equipment) all have thermal ratings and operation above these limits can cause permanent damage, regulatory issues or physical hazard. Because power is proportional to current, lines are normally given several different maximum apparent power operating limits. These include the familiar normal, continuous Rate A MVA equipment rating as well as the emergency, Rate C MVA limit. Given this, a moving average of the line flow is calculated and used for the transmission line thermal limit index function. This function is defined here as:

$$R_{TL}^l = min\left(1, \frac{P_{l,max} \cdot sl}{\sum_{r=t-sl}^{sl-1} P_{l,r}}\right) \tag{4}$$

from which the resilience measure of performance for transmission line thermal limits can be calculated as:

TABLE I
PARAMETERS FOR BUS VOLTAGE RESILIENCE FUNCTION

Symbol	Quantity	Value (pu)
V_{min}	Lower-Cut Out for Index Function	0.900
$V_{thr,min}$	Minimum Bus Voltage Value for Maximum Index Function	0.975
$V_{thr,max}$	Maximum Bus Voltage Value for Maximum Index Function	1.025
V_{min}	Upper-Cut Out for Index Function	1.100

$$Res_{R_{TL}^l,t} = \frac{\sum_{l=1}^{NL} max(0, R_{TL}^l(t) - R_{TL}^l(t-1))}{\sum_{l=1}^{NL} ceil(R_{TL}^l(t-1) - R_{TL}^l(t))} \tag{5}$$

C. Cooling water demand satisfaction resilience function

Thermoelectric generation requires water for the cooling cycle in the process of converting the heated working fluid from steam back to water via heat transfer. The closed-loop cooling cycle introduces losses through drift, blowdown and evaporation. So, water is needed to “make-up” for the amount lost and represents a key dependency of the power system on the water system. Generation owners meet this need of replenishing lost water through external sources such as a municipal water utility as well as by water that is stored on-site. Since the amount of onsite storage for cooling water is normally substantial [15], it is reasonable to assume that under normal conditions and even when operating under slightly sub-optimal conditions related to the supply of cooling water, the levels of the water in the onsite tanks will not change vary rapidly. As such, the system will still be able to function at a satisfactory level even with these small changes in onsite water storage level. The index function relating system performance to the amount of cooling water that is being supplied is proposed to be related to a conservative estimate of the storage tank level and given a piece-wise linear form as follows:

$$R_{CW}^g = \frac{ceil(\frac{TL_{g,t}}{TL_{g,initial}} \cdot 10)}{10} \tag{6}$$

The formulation of the index function in this way gives a slight lag in the functions value with respect to changes in the water storage tank level. This can be seen by examining how the value of R_{CW}^g changes as the tank level decreases. It is not until the tank level is under 90% of its initial value that the index function decreases, taking on a value of 0.9. This gives the desired behavior of tracking the tank level but also not changing too fast as the tank level experiences small fluctuations. The resilience measure of performance is then calculated using (6) as:

$$Res_{R_{CW}^g,t} = \frac{\sum_{g=1}^{NG} max(0, R_{CW}^g(t) - R_{CW}^g(t-1))}{\sum_{g=1}^{NG} ceil(1 - R_{CW}^g(t))} \tag{7}$$

Here, the difference between (7) as compared with (3) and (5) is seen in the denominator where the number of unsatisfactory performances in (7) is given a more strict definition.

In summary, this formulation quantifies the resilience with respect to cooling water as a piece-wise linear function based on the ratio between the current on-site cooling water to the initial tank level (based on a two week supply).

D. Demand supply satisfaction (overall) resilience function

A resilience measure related to the supply of electrical power demands of consumers is proposed in order to quantify how well the system demands were met under extreme system operating conditions where the possibility of load being shed or at least the necessity of additional power needing to be imported into the area under study was non-negligible. From the simulation results for the case studies under consideration, it was seen that with the implemented control strategy of derating units suffering cooling water shortages there would be times during which the total electrical demand would not be satisfied. In addition, it is known from consumer surveys [16] that they are willing to adjust their demands based on what the perceived weather conditions are. These consumers have also been shown to have slightly different amounts of willingness to change their consumption based on their knowledge of how the water and power system interact as well as whether the curtailment of water and energy usages is mandatory or not. For representation of this fact within the power system simulation, the bus loads are decomposed into representative percentages for industrial, commercial and residential components. Within the residential load component, it is further sub-divided into single and multi-family elements based on recent census data for Arizona [17]. The function for load supply satisfaction is then defined as follows:

$$R_{DS}^{n} = ceil\Big(\big(P_{load,nt}\text{-}LOL_{n,t}\big) - \big(P_{load,nt}\text{-}sf_{n} \cdot dAdj_{sf,t}\text{-}mf_{n} \cdot dAdj_{mf,t}\big)\Big) \tag{8a}$$

$$= ceil\left(sf_{n} \cdot dAdj_{sf,t} + mf_{n} \cdot dAdj_{mf,t}\text{-}LOL_{n,t}\right) \tag{8b}$$

This function is easily understood a term at a time. From (8a), the first term is the present bus load less any load that is lost and the second term is a best case load assuming the present demand is adjusted by a reduction equal to what consumers have said they would be willing to decrease consumption by. From this, the function is assigned a value of 1 if the actual load (less any lost) is greater than the best case, consumer adjusted load and zero if the opposite is true. The resilience measure of performance related to the supplied demand can then be calculated as:

$$Res_{R_{DS}^{n},t} = \frac{\sum_{n=1}^{NBL} max(0, R_{DS}^{n}(t)\text{-}R_{DS}^{n}(t\text{-}1))}{\sum_{n=1}^{NBL} ceil(R_{DS}^{n}(t\text{-}1)\text{-}R_{DS}^{n}(t))} \tag{9}$$

E. Load supply satisfaction (WDS pumps) resilience function

In an effort to further capture the interdependencies of the water and electric systems during operation, an additional index function is defined which is related to the satisfaction of the pump load directly serving the water distribution system. In contrast to the previous load metric, this one does not consider any consumer survey data but instead directly generates a one or a zero based on whether the pump demand is satisfied or not. The resilience function mathematically is given as:

$$R_{PL}^{n} = \begin{cases} 1, & PL_{demand} = PL_{supply} \\ 0, & PL_{demand} > PL_{supply} \end{cases} \tag{10}$$

The corresponding measure of performance is calculated as:

$$Res_{R_{PL}^{n},t} = \frac{\sum_{n=1}^{NPL} max(0, R_{PL}^{n}(t)\text{-}R_{PL}^{n}(t\text{-}1))}{\sum_{n=1}^{NPL} ceil(1\text{-}R_{PL}^{n}(t))} \tag{11}$$

F. Total System Resilience

The overall system resilience is then calculated as a weighted sum of the voltage, thermal limit, cooling water, demand supplied (overall) and demand supplied (WDS pumps) resilience values. The terms compose what are recognized as quantities pertaining solely to the operation of the power system, $f(Res_{PS,IND})$, as well as quantities concerned explicitly with the interdependent operation of the two systems, $f(Res_{PS,INTR})$. The weights can be determined through sensitivity analysis for the power system in order to judge the effect of each parameter in correctly assessing the overall performance of the system. The total resilience can be calculated at each time-period:

$$Res_{PS,TOT}(t) = f(Res_{PS,IND}) + f(Res_{PS,INTR}) \tag{12a}$$

$$Res_{PS,TOT}(t) = \frac{w_1 Res_{R_{V}^{n},t} + w_2 Res_{R_{TL}^{l},t} + w_3 Res_{R_{CW}^{g},t} + w_4 Res_{R_{DS}^{n},t} + w_5 Res_{R_{PL}^{n},t}}{w_1+w_2+w_3+w_4+w_5} \tag{12b}$$

IV. EXAMPLE OF PROPOSED METRICS AND A CASE STUDY

The following case study was conducted using a modified version of the IEEE 14 bus system. The modifications included the assigning of line limits based on the observed line flows over the course of many simulations with time-varying loads as well as representative distribution systems to which the WDS pump loads were modeled as being connected to. Fig. 2 shows the scenario definition for water shortages, representing drought conditions, as well as hours during which specific WDS pumps were assumed to be outaged. A complete description of the simulation setup, system parameters and scenario definition can be seen in [6].

V. RESULTS DISCUSSION

Fig. 3 contains the plots of the individual resilience measures of performance. The system is heavily loaded towards the beginning of the simulation and this is reflected in voltage and transmissions line resilience values that are less than 1. The restriction of water, emulating drought conditions, is seen to quickly degrade the cooling water measure of performance as the cooling water demand requirements of the thermoelectric generation are not met and the on-site storage tank levels begin to decrease. The demand satisfied measure of performance is seen to decrease later in the simulation as the implemented control scheme reduces the maximum power outputs of the units as the level of water in the on-site storage decreases. As mentioned in [6], this loss of load is an artifact of the small test system being used and can instead be interpreted as the amount of power needing to be imported for a large, interconnected system.

The plot in the bottom left of Fig. 3 shows the demand satisfied for the WDS pumps, which follows the trend in Fig. 2 in that these pumps are specified to be outaged in the scenario definition. The plot in the bottom right of Fig. 3 shows the components of the total resilience calculation in terms of the independent and interdependent measures of performance, as shown in Fig. 1 and defined in Section III.F. Lastly, Fig. 4 plots

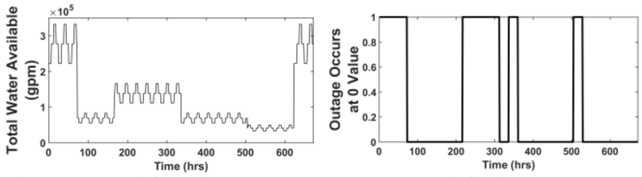

Fig. 2. Water (left) and power system contingency definitions versus time

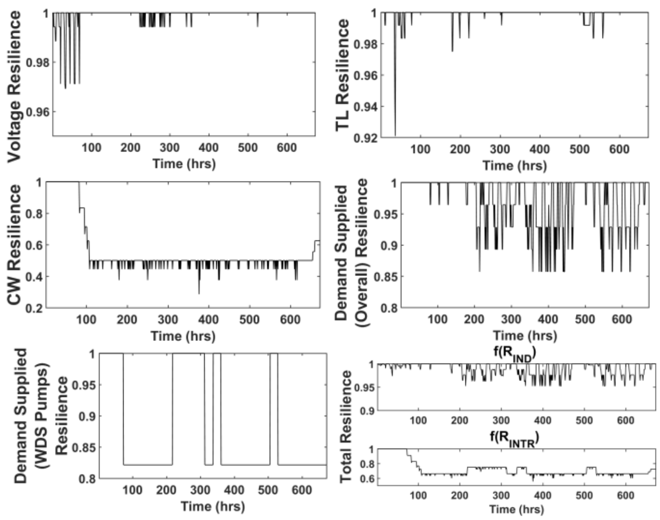

Fig. 3. Resilience measures of performance versus time. From L to R, Top to Bottom: resilience measure for voltage, thermal limits, on-site water storage level, load supplied (overall) and load supplied (WDS pumps)

the combined resilience metric for increasing weights for the terms capturing interdependent operation. Because these terms are the dominant component of the overall resilience value, it is seen that increasing values of these weights results in smaller decreases of the overall resilience function value. Lastly, it can be noted that selection of the specified indices results in capturing the desired system behavior, in that the measures of were observed. The combined resilience performance reflect the degradation in system conditions that value also reflects the fact that this test system is resilient in that the resilience value begins to increase at the end of the simulation. This is the result of no additional WDS pump outages within the power system and a larger volume of water available to the WDS to meet the thermoelectric generation demand in addition to its commercial and residential demands.

VI. Conclusion and Future Work

In this work, a novel formulation for the calculation of power system resilience and a methodology for this calculation involving operational measures of performance related to several quantities of interest in the power system were presented. The overall resilience is calculated with consideration of measures of performance which are explicit functions of power system operation only, including bus voltages and transmission line thermal limits, as well as measures of performance reflecting the interdependent water-energy system operation, consisting of on-site water storage tank levels and WDS pump loads being supplied electricity. The proposed method was then applied to a long term, time-domain simulation. Future work for improving this resilience calculation involves the modification of the weights in Equation (12b) to be time and scenario dependent in order to add the dimension of structural vulnerability which will allow the index functions to more accurately describe the system conditions.

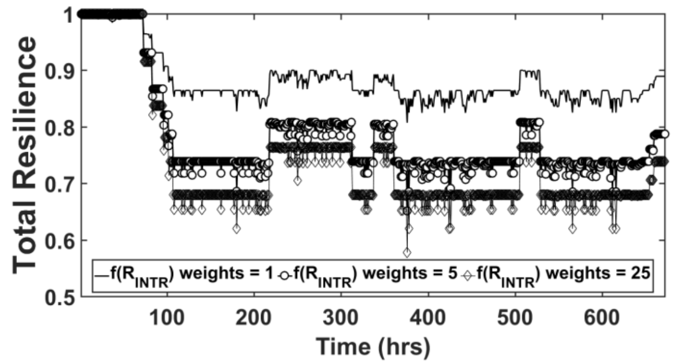

Fig. 4. Combined resilience function versus time for different interdependent system measure of performance weights

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